Cyber-physical systems under DoS attacks

A control systems perspective

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Outline

1. Switched systems
2. Cyber-Physical systems
3. Control design for CPS under attacks
4. Extensions and future directions
5. Final remarks

Switched systems

Consider the following discrete-time switched system

$$x(k+1) = A(\xi(k))x(k), \tag{1}$$

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$$A(\xi(k)) = \sum_{i=1}^{v} \xi_i(k)A_i = \xi_1(k)A_1 + \xi_2(k)A_2 + \ldots \xi_v(k)A_v, \tag{2}$$

and the indicator function is defined as

$$\xi_i(k) = \begin{cases} 1, & \text{for} \quad A_i \text{ (the } i\text{th mode is active)} \\ 0, & \text{otherwise} \end{cases} \qquad \forall i = 1, \ldots, v.$$

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Stability

How can we certify that system (1) is globally asymptotically stable?

Theorem

The zero equilibrium of $x(k+1) = f_k(x(k))$ *is globally uniformly asymptotically stable if there is a function* $V : \mathbb{Z}^+ \times \mathbb{R}^n \to \mathbb{R}$ *such that:*

- V *is a positive-definite function, decreasing along the trajectories, and radially unbounded;*
- $\Delta V(k, x(k)) = V(k+1, x(k+1)) - V(k, x(k))$ *is negative definite along the solutions of* $x(k+1) = f_k(x(k))$.

One can say that the Lyapunov function is positive-definite, decreasing along the trajectories, and radially unbounded if $V(k, 0) = 0, \forall k \geq 0$ *and*

$$\beta_1 \|x(k)\|^2 \leq V(k, x(k)) \leq \beta_2 \|x(k)\|^2 \tag{3}$$

for all $x(k) \in \mathbb{R}^n$ *and* $k \geq 0$ *with* β_1 *and* β_2 *positive scalars.*

Switched Lyapunov function[1] $V(k, x(k)) = x(k)^T P(\xi(k))x(k)$.

Theorem

If there exist symmetric matrices $P_1, \dots, P_v$, such that

$$\begin{bmatrix} P_i & A_i^T P_j \\ P_j A_i & P_j \end{bmatrix} > 0, \quad \forall (i, j) \in \mathcal{J} \times \mathcal{J}, \tag{4}$$

where $\mathcal{J} = \{1, \dots, v\}$, then, the Lyapunov function $V(k, x(k)) = x(k)^T P(\xi(k))x(k)$ certify the stability of the switched system $x(k+1) = A(\xi(k))x(k)$.

[1] J. Daafouz, P. Riedinger, and C. Iung, "Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach," IEEE Transactions on Automatic Control, 2002.

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Idea of the proof:

$$\begin{aligned} \Delta(V) &= V(k+1, x(k+1)) - V(k, x(k)) < 0 \\ &= x(k+1)^T P(\xi(k+1))x(k+1) - x(k)^T P(\xi(k))x(k) < 0 \\ &= x(k)^T (A(\xi(k))P(\xi(k+1))A(\xi(k)) - P(\xi(k)))x(k) < 0 \end{aligned}$$

[1] J. Daafouz, P. Riedinger, and C. Iung, "Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach," IEEE Transactions on Automatic Control, 2002.

Structured Lyapunov functions

By employing an augmented state vector in the Lyapunov function[2]

$$V(k) = \begin{bmatrix} x(k) \\ x(k+1) \\ \vdots \\ x(k+N-1) \end{bmatrix}^T \Psi \begin{bmatrix} x(k) \\ x(k+1) \\ \vdots \\ x(k+N-1) \end{bmatrix}$$

with

$$\Psi = \text{blkdiag}(P_1(\xi(k)), P_2(\xi(k+1)), \ldots, P_N(\xi(k+N-1))),$$

we are able to derive necessary and sufficient conditions to certify the stability of the switched system $x(k+1) = A(\xi(k))x(k)$.

[2] M. J. Lacerda and T. D. S. Gomide. "Stability and stabilisability of switched discrete-time systems based on structured Lyapunov functions". IET Control Theory & Applications, 2020.

Structured Lyapunov functions

The use of Lyapunov functions with non-monotonic terms[3] $V_i(x(k)) = x(k)^T P_i(\xi(k))x(k)$ can also lead to necessary and sufficient conditions[4] to certify the stability of the switched system $x(k+1) = A(\xi(k))x(k)$.

$$\sum_{i=j}^{N} V_i(x(k)) > 0, \quad j = 1, \ldots N,$$

$$V_1(x(k+1)) - V_1(x(k)) + V_2(x(k+2)) - V_2(x(k)) + \ldots + V_N(x(k+N)) - V_N(x(k)) < 0.$$

[3] A. A. Ahmadi and P. A. Parrilo. "Non-monotonic Lyapunov functions for stability of discrete time nonlinear and switched systems." 47th IEEE Conference on Decision and Control, 2008.

[4] M. J. Lacerda and T. D. S. Gomide. “Stability and stabilisability of switched discrete-time systems based on structured Lyapunov functions”. IET Control Theory & Applications, 2020.

Non-monotonic terms

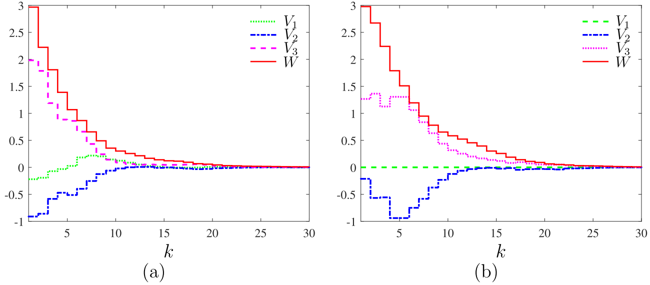

Fig. 1. Evolution of functions $V_1(x_k)$ (dotted green line), $V_2(x_k)$ (dashed dotted blue line), $V_3(x_k)$ (magenta dashed line) and the Lyapunov function $W(x_k)$ (straight red line) - Example 2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

M.L.C. Peixoto et al./Journal of the Franklin Institute 357 (2020) 6595–6614

Control design

Consider the following discrete-time switched system

$$x(k+1) = A(\xi(k))x(k) + B(\xi(k))u(k), \tag{5}$$

where $x \in \mathbb{R}^n$ is the state vector, and $u \in \mathbb{R}^{n_u}$ is the control input. The switching rule is unknown *a priori*, but it is considered to be available in real-time.

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State-feedback control

Design a switching state-feedback control law

$$u(k) = K(\xi(k))x(k),$$

where $K(\xi(k)) \in \mathbb{R}^{n_u \times n}$ stabilizes the closed-loop system

$$x(k+1) = (A(\xi(k)) + B(\xi(k))K(\xi(k)))x(k). \tag{6}$$

Theorem

If there exist symmetric matrices $P_j \in \mathbb{R}^{n\times n}$, $j = 1, \dots, N$, and matrices $X(\xi(k)) \in \mathbb{R}^{n\times n}$ and $Z(\xi(k)) \in \mathbb{R}^{n_u\times n}$, also defined as in (2), such that the following inequalities are satisfied

$$\sum_{m=j}^{N} P_m > 0, \quad j = 2, \dots, N$$

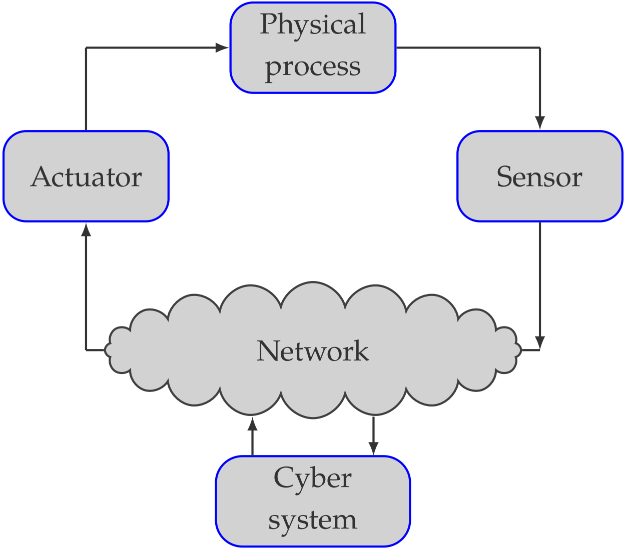
$$\begin{bmatrix} -P_1 & A_{i_1}X_{i_1} + B_{i_1}Z_{i_1} & \cdots & 0 & 0 \\ \star & P_1 - P_2 - X_{i_1} - X_{i_1}^T & \ddots & \vdots & \vdots \\ 0 & 0 & \ddots & A_{i_{N-1}}X_{i_{N-1}} + B_{i_{N-1}}Z_{i_{N-1}} & 0 \\ \vdots & \vdots & \ddots & P_{N-1} - P_N - X_{i_{N-1}} - X_{i_{N-1}}^T & A_{i_N}X_{i_N} + B_{i_N}Z_{i_N} \\ 0 & 0 & \cdots & \star & P_N - X_{i_N} - X_{i_N}^T \end{bmatrix} < 0$$

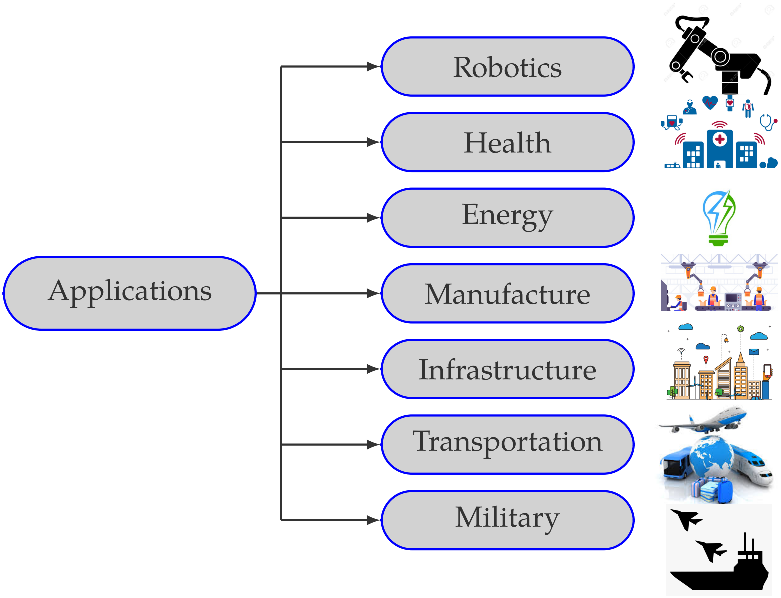
$$\forall (i_1, i_2, \dots, i_N) \in \underbrace{\mathcal{J} \times \mathcal{J} \dots \mathcal{J}}_{N \ times}$$

then, $K_{i_m} = Z_{i_m}X_{i_m}^{-1}$ are the state feedback control gains assuring that the closed loop system $x(k+1) = (A(\xi(k)) + B(\xi(k))K(\xi(k)))x(k)$ is asymptotically stable.

Cyber-Physical systems

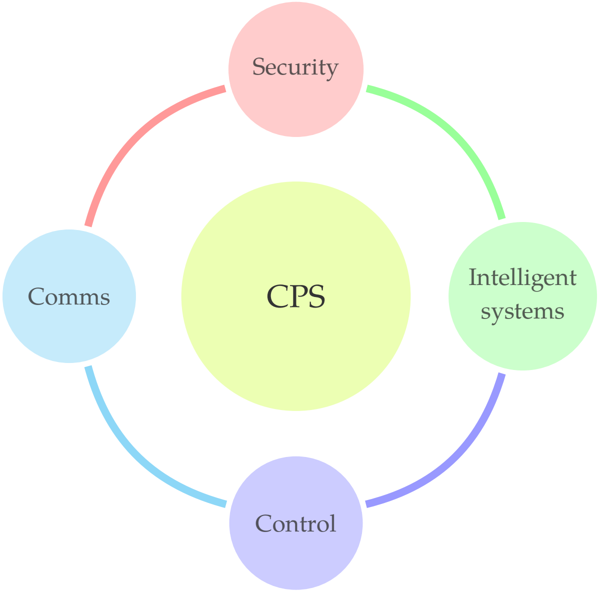
What is a cyber-physical system (CPS)?



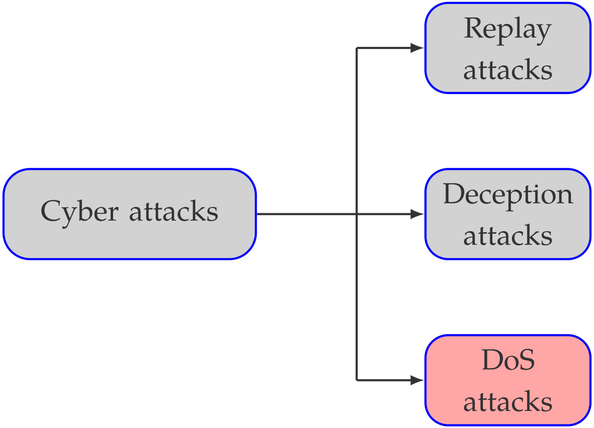

Applications
Robotics
Health
Energy
Manufacture
Infrastructure
Transportation
Military

Source: Data & Analytics Facility for National Infrastructure (DAFNI) to advance UK infrastructure research.

Virtual organization of a CPS

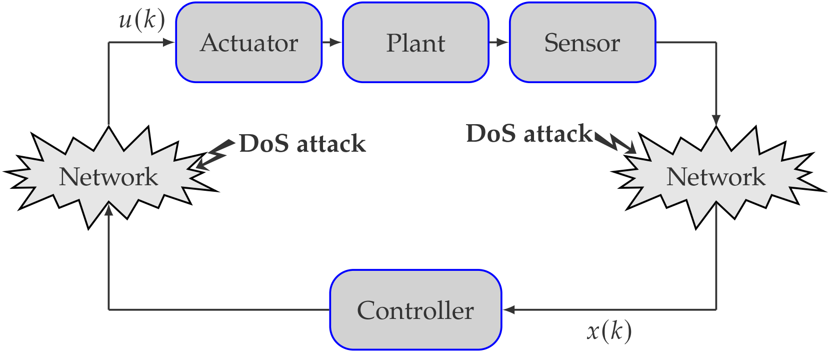


Cyber attacks on CPS[5]



[5]D. Ding, Q. L. Han, Y. Xiang, X. Ge, and X.-M. Zhang, "A survey on security control and attack detection for industrial cyber-physical systems," Neurocomputing, vol. 275, pp. 1674 – 1683, 2018.

Structure of a CPS under DoS attacks.



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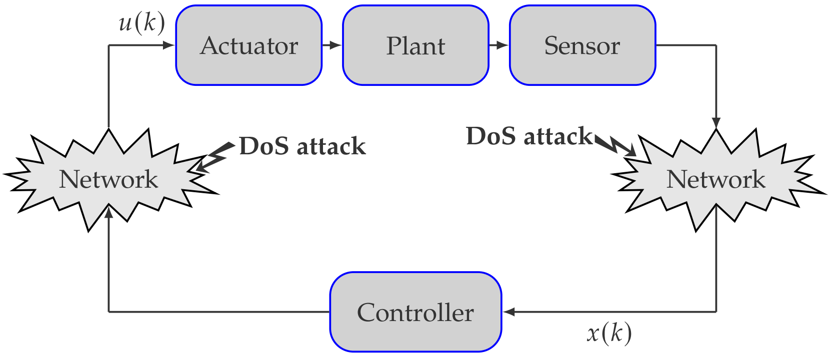

Problem

Does the designed controller ensure the stability of the closed-loop system under the presence of DoS attacks?

Control design for CPS under attacks

Control for cyber-physical systems

$$x(k+1) = A(\alpha)x(k) + B(\alpha)u(k) \tag{7}$$

where $x \in \mathbb{R}^n$ is the state vector, and $u \in \mathbb{R}^{n_u}$ the control input.

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Scenario

- The matrices $A(\alpha)$ and $B(\alpha)$ belong to an uncertain domain.

$$\begin{bmatrix} A(\alpha) & B(\alpha) \end{bmatrix} = \sum_{i=1}^{V} \alpha_i \begin{bmatrix} A_i & B_i \end{bmatrix}, \quad \alpha \in \Lambda,$$

- V denotes the number of vertices of the polytope and Λ is the unit simplex

$$\Lambda = \left\{ \alpha \in \mathbb{R}^V : \sum_{i=1}^{V} \alpha_i = 1, \alpha_i \geq 0 \right\}.$$

Motivation

Consider a discrete-time uncertain system with matrices

$$A = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 - 0.1\delta \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0.1\kappa \end{bmatrix},$$

where $0.1s^{-1} \leq \delta \leq 10s^{-1}$, and $\kappa = 0.787rad^{-1}V^{-1}s^{-2}$.

Disregarding the existence of attack the following state-feedback control gain stabilizes the system

$$K = \begin{bmatrix} -6.6145 & -7.4944 \end{bmatrix}.$$

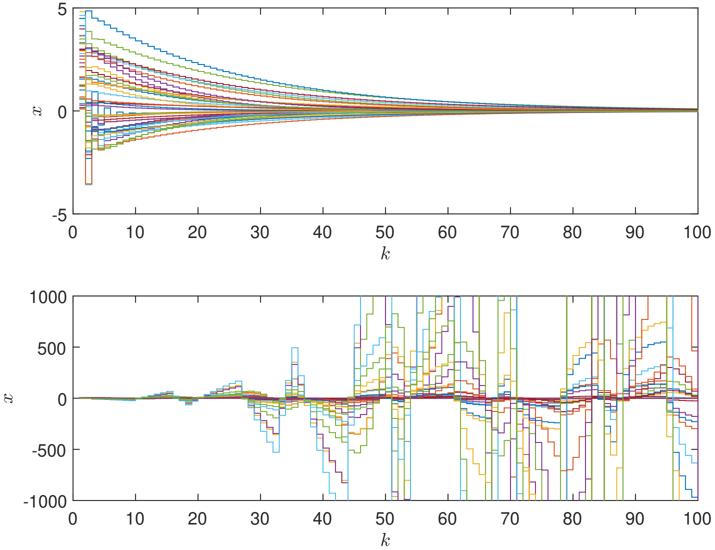


Figure 1: Trajectories for the closed-loop states in the absence of attacks (upper), and during the presence of DoS attack (lower).

Problem

How can we design a control strategy capable of ensuring the stability of the closed-loop uncertain system under the presence of DoS attacks?

- We need to construct a model that takes into account the presence of DoS attacks. Different control strategies can be employed[6]:
  - Hold strategy
  - Zero strategy
  - Packet of different controllers
- By using the Lyapunov theory, the design conditions will be written in the form of LMIs.
- The designed controllers will be capable of ensuring the stability of the closed-loop uncertain system under the presence of DoS attacks.

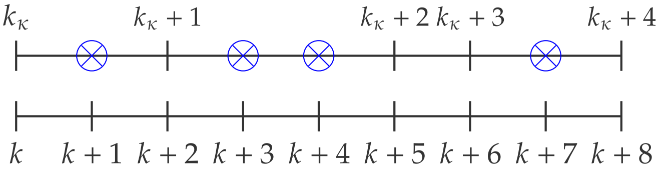
[6] L. Schenato, "To Zero or to Hold Control Inputs With Lossy Links?," IEEE Transactions on Automatic Control, 2009.

Problem Formulation: DoS modelling

Assumption: The duration of the DoS attack is bounded by the maximum number of consecutive control inputs samples that do not get to the actuator, being this number denoted by N.

- Switching signal $\sigma(k_\kappa)$ that assume values in $M \triangleq \{0, 1, \ldots, N\}$
- A new time scale k_κ that represents the time instant when the updated control input reaches the actuator

$$k_\kappa + 1 = k_\kappa + \sigma(k_\kappa) + 1, \quad k_0 = 0, \quad \sigma(k_\kappa) = \{1, 2, 0, 1\}$$



Problem Formulation: Hold Strategy

- The same control input $u(k) = Kx(k)$ available to the actuator is successively applied until the end of the attack (next successful transmission).

$N = 1$

$$x(k+1) = A(\alpha)x(k) + B(\alpha)Kx(k),$$

$$x(k+2) = A(\alpha)x(k+1) + B(\alpha)Kx(k),$$

$$\rightarrow x(k+2) = A(\alpha)^2 x(k) + A(\alpha)B(\alpha)Kx(k) + B(\alpha)Kx(k),$$

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$N = 2$

$$x(k+3) = A(\alpha)x(k+2) + B(\alpha)Kx(k),$$
$$\rightarrow x(k+3) = A(\alpha)^3x(k) + A(\alpha)^2B(\alpha)Kx(k) + A(\alpha)B(\alpha)Kx(k) + B(\alpha)Kx(k).$$

Problem Formulation: Zero Strategy

- The control input is set to zero until the end of the attack (next successful transmission).

$N = 1$

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\begin{aligned}
x(k+1) &= A(\alpha)x(k) + B(\alpha)Kx(k), \\
x(k+2) &= A(\alpha)x(k+1) \\
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\end{aligned} \tag{8}
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$$

Problem Formulation: Packet Strategy

- Different control inputs $u(k+i) = K_i\, x(k)$ are available to the actuator before an attack starts in $k+1$. These inputs are successively applied until the end of the attack (next successful transmission).

$N = 1$

$$
\begin{aligned}
x(k+1) &= A(\alpha)x(k) + B(\alpha)K_0 x(k),\\
x(k+2) &= A(\alpha)x(k+1) + B(\alpha)K_1 x(k),\\
\rightarrow x(k+2) &= A(\alpha)^2 x(k) + A(\alpha)B(\alpha)K_0 x(k) + B(\alpha)K_1 x(k),
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Packet-based approach[7]

Packet of controllers

$$U(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+N) \end{bmatrix} = \begin{bmatrix} K_0 x(k) \\ K_1 x(k) \\ \vdots \\ K_N x(k) \end{bmatrix}, \tag{9}$$

is the package that gets to the actuator side every time that the communications channels are free of the attack.

[7] P. S. P. Pessim and M. J. Lacerda, "State-Feedback Control for Cyber-Physical LPV Systems Under DoS Attacks." *IEEE Control Systems Letters*, 2021.

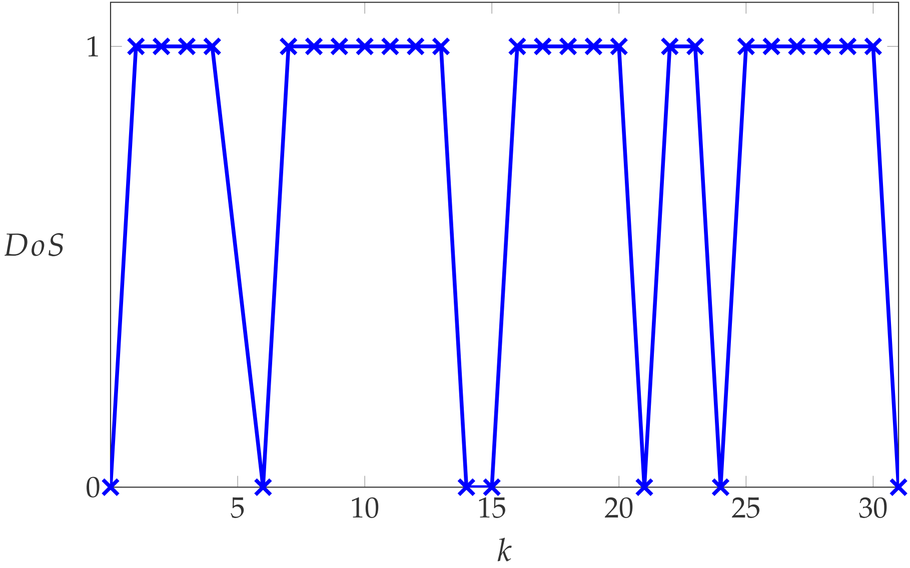

Figure 2: "1" - presence of DoS attacks and "0" - absence of DoS attacks. Sequence of attacks $\sigma(k_\kappa) = \{4, 7, 0, 5, 2, 6, \ldots\}$

$$U_1(k_\kappa) = \begin{bmatrix} u(k_\kappa) \\ u(k_\kappa+1) \\ u(k_\kappa+2) \\ u(k_\kappa+3) \\ u(k_\kappa+4) \\ u(k_\kappa+5) \\ u(k_\kappa+6) \\ u(k_\kappa+7) \end{bmatrix}, \; U_2(k_\kappa) = \begin{bmatrix} u(k_\kappa) \\ u(k_\kappa+1) \\ u(k_\kappa+2) \\ u(k_\kappa+3) \\ u(k_\kappa+4) \\ u(k_\kappa+5) \\ u(k_\kappa+6) \\ u(k_\kappa+7) \end{bmatrix}, \; U_3(k_\kappa) = \begin{bmatrix} u(k_\kappa) \\ u(k_\kappa+1) \\ u(k_\kappa+2) \\ u(k_\kappa+3) \\ u(k_\kappa+4) \\ u(k_\kappa+5) \\ u(k_\kappa+6) \\ u(k_\kappa+7) \end{bmatrix},$$

$$U_4(k_\kappa) = \begin{bmatrix} u(k_\kappa) \\ u(k_\kappa+1) \\ u(k_\kappa+2) \\ u(k_\kappa+3) \\ u(k_\kappa+4) \\ u(k_\kappa+5) \\ u(k_\kappa+6) \\ u(k_\kappa+7) \end{bmatrix}, \; U_5(k_\kappa) = \begin{bmatrix} u(k_\kappa) \\ u(k_\kappa+1) \\ u(k_\kappa+2) \\ u(k_\kappa+3) \\ u(k_\kappa+4) \\ u(k_\kappa+5) \\ u(k_\kappa+6) \\ u(k_\kappa+7) \end{bmatrix}, \; U_6(k_\kappa) = \begin{bmatrix} u(k_\kappa) \\ u(k_\kappa+1) \\ u(k_\kappa+2) \\ u(k_\kappa+3) \\ u(k_\kappa+4) \\ u(k_\kappa+5) \\ u(k_\kappa+6) \\ u(k_\kappa+7) \end{bmatrix}.$$

Problem Formulation: Switched System

- Case 0: DoS-free case

$$x(k_\kappa + 1) = (A(\alpha) + B(\alpha)K_0)\, x(k_\kappa),$$
$$x(k_\kappa + 1) = F_0(\alpha)x(k_\kappa)$$

- Case 1: The DoS attack occurs during one time-instant

$$x(k_\kappa + 1) = \left(A(\alpha)^2 + A(\alpha)B(\alpha)K_0 + B(\alpha)K_1\right) x(k_\kappa),$$
$$x(k_\kappa + 1) = F_1(\alpha)x(k_\kappa) = (A(\alpha)F_0(\alpha) + B(\alpha)K_1)\, x(k_\kappa).$$

- Case 2: The DoS attack occurs during two time-instants

$$x(k_\kappa + 1) = F_2(\alpha)x(k_\kappa) = (A(\alpha)F_1(\alpha) + B(\alpha)K_2)\, x(k_\kappa).$$

Problem Formulation: Switched System

A generic formulation is given as follows

$$F_i(\alpha) = A(\alpha)F_{i-1}(\alpha) + B(\alpha)K_i,$$

$i = 1, \ldots, N$, with $F_0(\alpha) = A(\alpha) + B(\alpha)K_0$. These matrices are used to construct the following switched system with $N+1$ modes.

$$x(k_\kappa + 1) = F_{\sigma(k_\kappa)} x(k_\kappa).$$

Considering the indicator function $\xi(k_\kappa) = [\xi_0(k_\kappa), \ldots, \xi_N(k_\kappa)]^\top$

$$x(k_\kappa + 1) = F(\xi(k_\kappa))x(k_\kappa), \quad \xi_i(k_\kappa) = \begin{cases} 1, & \text{if } \sigma(k_\kappa) = i \\ 0, & \text{otherwise} \end{cases}$$

with $F(\xi(k_\kappa)) = \xi_0(k_\kappa)F_0 + \xi_1(k_\kappa)F_1 + \cdots + \xi_N(k_\kappa)F_N$.

How to design the gain matrices?

Existence of a Lyapunov function $V(x_{k_\kappa})$, that is positive definite, and has its time rate of change negative definite along the trajectories, i.e., $\Delta V(x_{k_\kappa}) < 0$.

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Moreover, we need to employ

1. Change of variables.
2. Congruence transformation.
3. Schur complement.
4. Linear Matrix Inequalities.

Theorem

If there exist symmetric positive definite matrices $Q_i \in \mathbb{R}^{n\times n}$, *matrices* $X \in \mathbb{R}^{n\times n}$ *and* $Z_i \in \mathbb{R}^{n_u\times n}$, *such that*

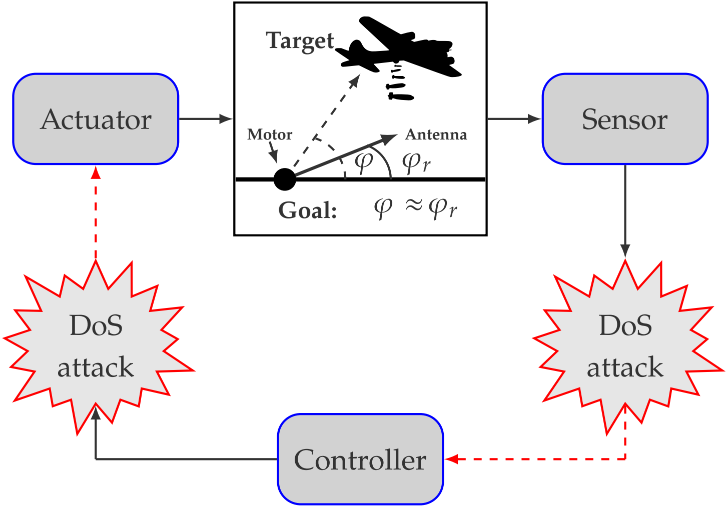
$$\begin{bmatrix} -Q_i(\alpha) & \star \\ \Psi_i & Q_j(\alpha) - X - X^T \end{bmatrix} < 0, \tag{10}$$

where

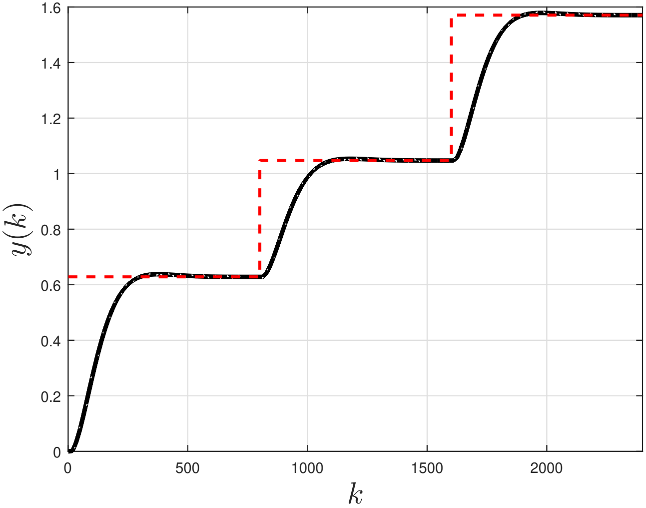
$$\Psi_i = A(\alpha)^{i+1} X + \sum_{m=0}^{i} A(\alpha)^m B(\alpha) Z_{i-m}, \tag{11}$$

with $A(\alpha)^0 = I_n$, $i, j \in M$, $M \triangleq \{0, 1, \ldots, N\}$, *then* $K_i = Z_i X^{-1}$ *are the state-feedback control gains that assure the closed-loop system (7) is asymptotically stable.*

Example: angular positioning system



Example: angular positioning system $N = 14$



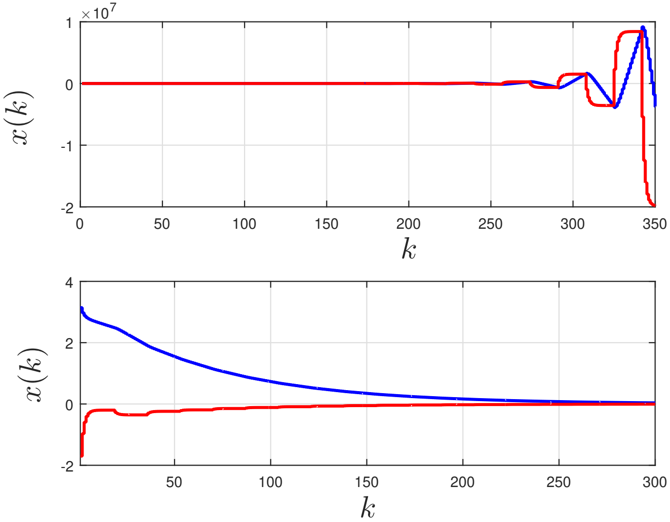


Figure 3: Trajectories of the system considering the presence of attack $N = 16$ in the design conditions (bottom) and disregarding the presence of attack in control design (top).

Problems addressed

- Output-feedback control for LPV systems[8].
- $\mathscr{H}_\infty$ performance for LPV systems[9].
- $\mathscr{H}_2$ performance for uncertain systems[10].

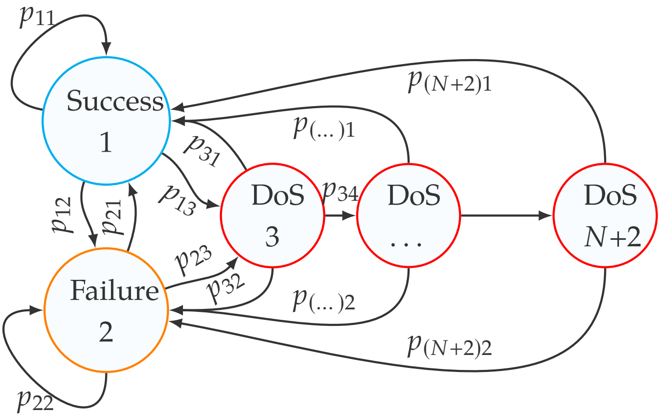
[8] P. S. P. Pessim and M. L. C. Peixoto and R. M. Palhares and M. J. Lacerda, "Static output-feedback control for Cyber-physical LPV systems under DoS attacks."*Information Sciences*, 2021.

[9] P. S. P. Pessim and M. J. Lacerda, "On the robustness of Cyber-physical LPV systems under DoS attacks." *Journal of the Franklin Institute*, 2022.

[10] P. M. Oliveira and J. M. Palma and M. J. Lacerda. "$\mathscr{H}_2$ state-feedback control for discrete-time cyber-physical uncertain systems under DoS attacks," *Applied Mathematics and Computation*, 2022.

Extensions and future directions

A model that includes DoS attack+packet loss for control design[11].



[11] P. M. Oliveira and J. M. Palma and M. J. Lacerda. " Control Design for an Unreliable Markovian Network Susceptible to Denial-of-Service Attacks", IEEE Transactions on Circuits and Systems II: Express Briefs, 2024.

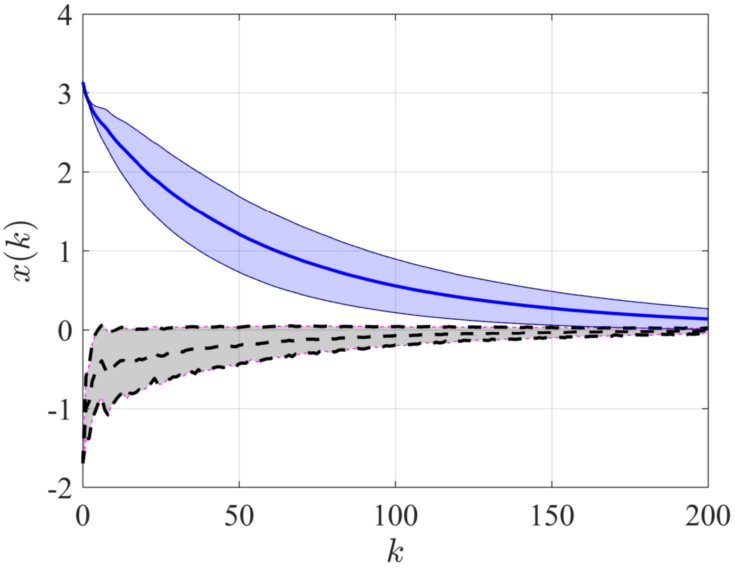
Consider the same example with

$$A(\alpha) = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 - 0.1\delta \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.1\kappa \end{bmatrix},$$

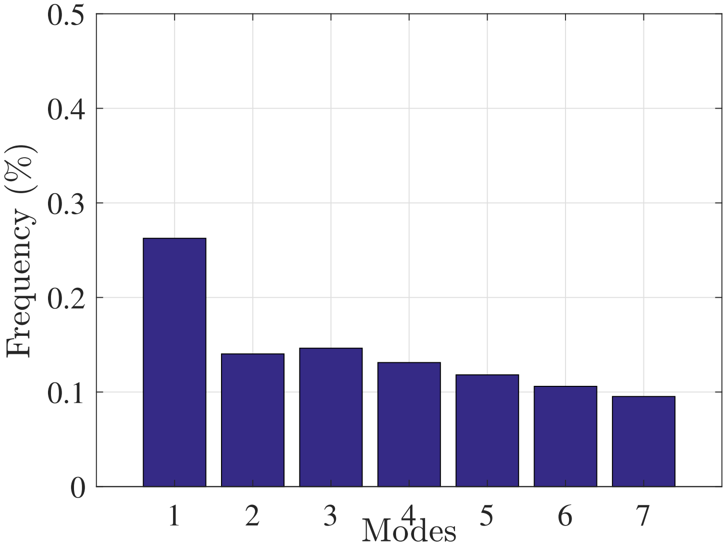
where $0.1s^{-1} \leq \delta \leq 10s^{-1}$, and $\kappa = 0.787 rad^{-1}V^{-1}s^{-2}$. In this approach we need to take into account the transition probability matrix.

$$\Psi = \begin{bmatrix} 0.5 & c & d & 0 & 0 & 0 & 0 \\ 0.4 & ? & ? & 0 & 0 & 0 & 0 \\ 0.05 & 0.05 & 0 & 0.9 & 0 & 0 & 0 \\ 0.05 & 0.05 & 0 & 0 & 0.9 & 0 & 0 \\ 0.05 & 0.05 & 0 & 0 & 0 & 0.9 & 0 \\ 0.05 & 0.05 & 0 & 0 & 0 & 0 & 0.9 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{12}$$

where $c = \begin{bmatrix} 0.05 & 0.15 \end{bmatrix}$ and $d = \begin{bmatrix} 0.35 & 0.45 \end{bmatrix}$.



4
3
2
1
0
-1
-2
$x(k)$
0
50
100
150
200
k


0.5
0.4
0.3
0.2
0.1
0
Frequency (%)
1
2
3
4
5
6
7
Modes

Perspectives for future research

Secure control

- Safety for cyber-physical systems under attacks using control barrier function.
- Filter design for attack detection.
- Constrained control input such as saturation.
- Replay attacks and false data injection attacks.
- Hybrid model for the CPS under attack.

Final remarks

Emerging Methodologies

- Safety Critical systems
- Resilient cyber-physical systems
- Cyber-physical human systems

Summary

- CPS present opportunities and new challenges for control design.
- Control theory can contribute to safety in CPS.

Summary

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- Control theory can contribute to safety in CPS.

Thank you!

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